Zuse Institute Berlin (ZIB)
Freie Universität Berlin
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5. Homework
Numerics for Bioinformaticians
WS 2016/17

Deadline: November 30, 10:00 (**before** the lecture)

The homework should be worked out in groups of two or three students. Each solution sheet must contain the names and student numbers ('Matrikulationnummer') of all group members and the exercise group (Wednesday/Friday). Solutions to homework 1&2 must be handed in in paper form, either hand written or printed out if generated electronically. Please staple all sheets. Programming tasks must be submitted to max.kleist2@fu-berlin.de or BioInfNumerik@hotmail.com by email. Before sending it, please 'zip' it.

Exercise 1 (Modelling, 2 points)
You are given the following model:

$$\begin{array}{llll} R_1: & x_1 + x_2 \xrightarrow{k_1} \emptyset & R_2: & x_1 + x_4 \xrightarrow{k_2} x_2 + x_3 \\ R_3: & x_1 + x_2 \xrightarrow{k_3} x_1 + x_2 & R_4: & x_2 + x_3 \xrightarrow{k_4} x_2 + x_4 \\ R_5: & \emptyset \xrightarrow{k_5} x_4 & R_6: & x_2 + x_3 + x_4 \xrightarrow{k_5 + k_4} x_3 + x_4 \end{array}$$

write down the corresponding ordinary differential equations (ODE).

Exercise 2 (Integration, 2 points)
Using Proposition 4 show that,

$$\int_0^{\Delta x} e^x dx = e^{\Delta x} - 1.$$

Exercise 3 (Implementation: Integration, 1 + 2 + 1 + 2 points)
Hasenheide. The following ODE is often used to describe the pharmacokinetics (concentration-time profile) of pharmaceutic drugs X in the bloodstream after a single dose D intake:

$$f(t) = \frac{\mathrm{d}}{\mathrm{d}t}x = k_a \left(\frac{D}{V} \cdot e^{-k_a \cdot t} \right) - k_e \cdot x(t) \qquad (1)$$

where k_a and k_e are parameters describing the uptake and elimination of the drug from the body. In our example, let $k_e = 0.3$ and $k_a = 0.5$ and let the dosage of the drug be $D = 200$ [mg] and let the initial concentration in the bloodstream be $x_0 = 0$. For simplicity, let $V = 1$ [L] and thus X be units of [mg/L].
Compute the integral of equation (**??**)

$$\tilde{x}(t + \Delta t) = x(t) + \int_t^{t+\Delta t} f(t) ds$$

for $t = (0, \Delta t, 2\Delta t, \ldots t_{final} - \Delta t)$, where $t_{final} = 24$, ...
a)... using the rectangle rule with fixed step size $\Delta t = 1$.
Plot your integration result using the rectangle rule at each time instance $(0, \Delta t, 2\Delta t, \ldots, 24)$. For comparison, put the exact solution for these time instances using eq. (**??**) into the same plot.
Hint 1: Use eq. (**??**) during integration (in the equation above and in the right hand side of eq. (**??**)).

Hint 2: An example of how this should look like is found in Fig. **??** (left).

b) Try $\Delta t = 0.1,\ 0.5,\ 1, 2, 3$ and compute the average absolute difference between your solution and the exact solution, i.e.

$$\varepsilon = \frac{1}{N}\sum |\tilde{x}(t) - x(t)|$$

where $N = t_{final}/\Delta t$ denotes the number of time steps. Plot ε (y-axis) against Δt (x-axis) on a log-log scale (Example: Fig. **??**, right)

c) Compute the integral of equation (**??**)

$$\tilde{x}(t + \Delta t) = x(t) + \int_t^{t+\Delta t} f(t)ds$$

for $t = (0, \Delta t, 2\Delta t, \ldots t_{final} - \Delta t)$, where $t_{final} = 24$, using the trapezoidal rule with fixed step size $\Delta t = 1$. Plot your integration result using the mid-point rule at each time instance $(0, \Delta t, 2\Delta t, \ldots, 24)$. For comparison, put the exact solution for these time instances using eq. (**??**) into the same plot.

Hint 1: Use eq. (**??**) during integration (in the equation above and in the right hand side of eq. (**??**)).

Hint 2: An example of how this should look like is found in Fig. **??** (left).

d) Try $\Delta t = 0.1,\ 0.5,\ 1, 2, 3$ and compute the average absolute difference between your solution and the exact solution, i.e.

$$\varepsilon = \frac{1}{N}\sum |\tilde{x}(t) - x(t)|$$

where $N = t_{final}/\Delta t$ denotes the number of time steps. Plot ε (y-axis) against Δt (x-axis) on a log-log scale and put it into the same plot as b), see Fig. **??** (right) for an example.

Exact solution: The exact solution to equation (**??**) is given by

$$x(t) = D\left(\frac{k_a}{k_a - k_e}\right)\left(e^{-k_e \cdot t} - e^{-k_a \cdot t}\right). \tag{2}$$

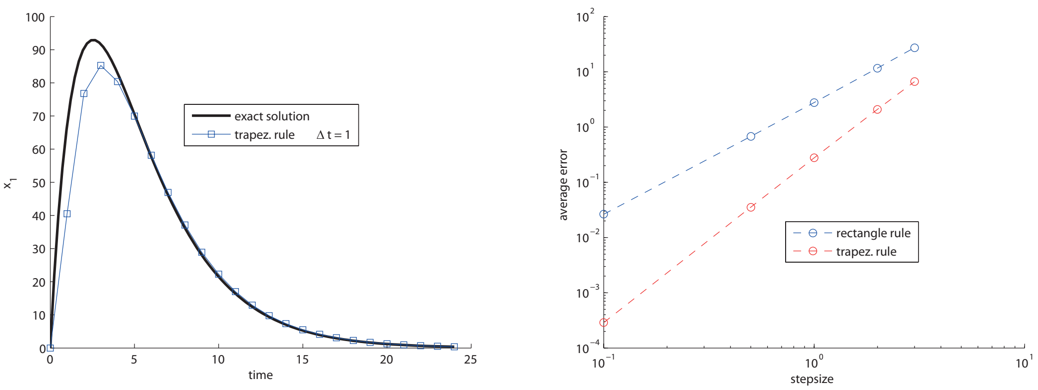

Figure 1: Left:How to plot in Ex.3a & 3c. Right: How to plot in Ex.3b & 3d