Zuse Institute Berlin (ZIB)
Freie Universität Berlin
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(Optional) numerical christmas/new years tasks
Numerics for Bioinformaticians
WS 2016/17

Deadline: January 4, 10:00 (**before** the lecture)

The homework should be worked out in groups of two or three students. Each solution sheet must contain the names and student numbers ('Matrikulationnummer') of all group members and the exercise group (Wednesday/Friday). Solutions to homework 1&2 must be handed in in paper form, either hand written or printed out if generated electronically. Please staple all sheets. Programming tasks must be submitted to max.kleist2@fu-berlin.de or BioInfNumerik@hotmail.com by email. Before sending it, please 'zip' it.

Exercise 1 (The damn rabbit and the weird medicine, 2+1+2 points)
It's **seven days** to Christmas and the whole north-pole crowd, including Santa, enjoy themselves too much. At some point Santa falls asleep. During this time, the nasty easter-bunny shaved off his beard! Only **a single hair** is left! Looking into the mirror (below), Santa is sure he cannot present himself to the children (they may get a trauma)! Chaos is breaking loose at the north pole and meanwhile the UN (united northpoleans) is thinking of cancelling the entire event.
What to do? Under normal conditions Santa will grow back **3** hairs of his beard **per day**, but this will not suffice to get the total amount of 999 hairs back by christmas. Luckily, one of the Christmas gnomes has some dubious contact and can get an unapproved (off-license) m(ed/ag)ical product. According to the package insert, this product suppresses the normal beard growth, but **triples the number of existing hair each time** it is applied. However, it can't be applied more than once a day, otherwise Santa becomes as evil as the easter-bunny. Therefore it is quite important to use precision medicine here: I.e. not to under- or overdose the medicine and to make sure Santa has **exactly 999** hears in his beard in 7 days; i.e. there are seven decisions to make that could save christmas....(or ruin it)!
a) Santa has initially 1 hair. On which of the seven days does Santa have to take the 'medicine' to gain his gorgeous beard with exactly 999 hears back?
b) If you come up with an algorithm that guarantees that you find the solution using less than 2^7 steps you get an extra point. (you will have to justify the complexity of the algorithm).
c) If you come up with an algorithm that finds the solution in 28 calculation steps or less you get all 5 points.
Hint: Think backwards.
Background: The task is a so-called *optimal control problem*, which is another branch of *numerics* that deals with controlling devices (GPS, rotating of the display in the smartphone, ... and hopefully *precision medicine* one day...)

Exercise 2 (New years eve party, 1+2+1+1 points)
You are throwing a new years eve party in your 38qm flat and didn't invite your numerics lecturers (burn!). All in all $x_0 = 300$ will come (friends and friends of friends and random people you never saw before and after and during). The party evolves as follows: people come (R_1) and go (R_2).

$$R_1: \quad x_0 \xrightarrow{k_{\text{in}}} x_1 \tag{1}$$

$$R_2: \quad x_1 \xrightarrow{k_{\text{out}}} \emptyset \tag{2}$$

where x_0 are the individuals that have not been at your party yet and x_1 are the number of individuals that are at your party. Parameters are $k_{\text{in}} = 0.4$ and $k_{\text{out}} = 0.3$ in units [1/h].
a) Write down the ODEs for this model.
b) Solve the ODE analytically.
Hint: It should look familiar.
c) You start your party at 8pm. When do you expect to have most people in your apartment?
d) How many individuals do you expect at your apartment at maximum? And how many square meters will there be for each person?
Hint (c+d): The ODE gives you the expected number of individuals.

Figure 1: The protagonists of Exc. 1...