Zuse Institute Berlin (ZIB)
Freie Universität Berlin
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11. Homework
Numerics for Bioinformaticians
WS 2016/17

Deadline: January 25, 10:00 (**before** the lecture)

The homework should be worked out in groups of two or three students. Each solution sheet must contain the names and student numbers ('Matrikulationnummer') of all group members and the exercise group (Wednesday/Friday). Solutions to exercise 1.b & 3 must be handed in in paper form, either hand written or printed out if generated electronically. Please staple all sheets. Programming tasks must be submitted to max.kleist2@fu-berlin.de or BioInfNumerik@hotmail.com by email. Before sending it, please 'zip' it.

Exercise 1 (Poisson Likelihood, 4 points)
Consider the structural model $\mathcal{M}$:

$$x(t_i) = \frac{\lambda}{\delta}\left(1 - e^{-\delta t_i}\right).$$

The likelihood of observing y_i at t_i for a parameter choice $\lambda, \delta > 0$, is given by,

$$\mathcal{L}(y_i, t_i|\lambda, \delta) = \frac{x(t_i)^{y_i} e^{-x(t_i)}}{y_i!}.$$

Given the measurements are $y = \{108, 108, 101, 108, 109, 91, 108, 97, 92, 98\}$, all taken at the same time $t = 100$.

a) Plot a contour plot of the likelihood, for the given data, with axis $\lambda = [0.1, 30]$ and $\delta = [0.01, 0.3]$.

b) On the contour plot, plot the line $\{(\lambda, 0.102 \times \lambda)|\lambda \in [0.1, 30]\}$. What can you say about the region of maximum?

Exercise 2 (Optimisation Package, 2 Points)
Consider the structural model $\mathcal{M}$:

$$x(t_i) = D\left(\frac{k_a}{k_a - k_e}\right)\left(e^{-k_e \cdot t_i} - e^{-k_a \cdot t_i}\right)$$

with the dose $D = 200$ [mg], assuming an additive error

$$y_i = x_{i|k_e,k_a} + \eta_i$$

where $\eta_i \sim \mathcal{N}(0, \sigma_i^2)$.
With $\sigma^2 = \{4, 4, 16, 4, 1, 4\}$. The measurements are $y = \{48.52, 70.61, 82.57, 72.87, 35.09, 7.37\}$ for $t = \{1, 2, 5, 7, 12, 24\}$ [h] post dosing.
1) In your respective programming language, use an off the shelf minimisation function to find k_e, k_a which minimise the weighted least squares. Verify your result with the corresponding contour plot you generated in Homework 10.

Exercise 3 (Fixed Points, Points 4)
Consider the adjusted Slögl model,

$$\frac{dx_1(t)}{dt} = k_1 x_1^2(t) - k_2 x_1^3(t) - k_3 x_1(t),$$

with parameter values are $0.5, 0.015$ and 3.5 for k_1, k_2, k_3 respectively.

a) Compute the fixed points and their respective dynamical behaviour.